Classical Kinetic Theory of Landau Damping for Self-interacting Scalar Fields in the Broken Phase

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October 8, 2018

Abstract

The classical kinetic theory of one-component self-interacting scalar fields is formulated in the broken symmetry phase and applied to the phenomenon of Landau damping. The domain of validity of the classical approach is found by comparing with the result of a 1-loop quantum calculation.

1 Introduction

For high enough temperature leading non-equlibrium transport effects of large wave number $|\mathbf{k}| >> T$ fluctuations can be reproduced by a kinetic theory of the corresponding response functions. This approach has been applied to the features of the QCD plasma [1] and proved successfull in reproducing the contribution of hard loops to the Green functions of low $|\mathbf{k}| << T$ modes in Abelian and non-Abelian gauge theories [2, 3].

These investigations have assumed tacitly that no background fields are present, all symmetries are restored. This assumption is certainly not valid

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for all field theoretical models, since in some scalar models with specific internal symmetries one finds arguments in favor of non-restoration of the symmetry at arbitrary high temperature [4, 5]. Also certain approaches to non-Abelian gauge+Higgs systems provide evidence for non-zero scalar expectation values even in the high-temperature phase [6]. Therefore, there exist intuitive hints for possible relevance of the classical kinetic considerations in a non-zero scalar background even at high temperature.

The classical kinetic theory for self-interacting scalar fields has been derived first by Danielewicz and Mrówczynski [7] and its features are still being discussed [8]. These papers deal with scalar theories of non-negative squared mass parameter. Therefore their results are useful for the calculation of transport characteristics (plasmon frequency, damping rates) in the restored symmetry phase. In Ref. [8] an effective action is derived, which accounts for the loop contribution of high-k fluctuations to the Green functions of the low-k modes. As a parallel evolution one may note that a classical cut-off field theoretic approach to time dependent phenomena in the symmetric phase has been developed in Refs. [9, 10, 11, 12]. Quantitative results relevant for quantum systems were obtained by matching the classical theory to the parameters of the quantum theory. This effective theory reproduces, for instance, correctly the on-shell damping rate in the high temperature phase.

The present note rederives in a fully relativistic Lagrangian formalism the kinetic theory of scalar fields for the case of non-zero background field (generated by a negative mass squared parameter). It starts by proposing a Lagrangian for the effective particles, which accounts for the effect of the high-k φ modes on the low-k fluctuations. A detailed motivation for this Lagrangian is provided by recalling results of earlier investigations. An advantage of this proposition is that it leads to the induced source density of the low-k fluctuations directly, without any reference to the quantum theory. Evidence for the correctness of the effective lagrangian can be presented by comparing its consequences with the results of the corresponding quantum calculations. As a first test we compute a physically meaningful quantity, the Landau damping coefficient for the off-shell scalar fluctuations. This effect is present only in the broken symmetry phase of the scalar theory. Its classical value is compared with the result of a 1-loop quantum calculation [13], establishing in this way the domain of validity of the proposed classical treatment.

2 Kinetic theory of the high-k modes in a non-zero background

The effective gas of high-frequency fluctuations is out of thermal equilibrium if an inhomogenous low frequency background fluctuation is present. This state of the gas induces a source term into the wave equation of the low-k modes. A unified description of the two effects can be attempted if in addition to the action S_{cl} describing the low-frquency dynamics an appropriate Lagrangian can be introduced for a gas particle coupled to the low-frequency field along its trajectory $\xi_\mu(\tau)$:

$$\begin{aligned} S_{eff} &= S_{cl}[\varphi] + \Delta S, \\ \Delta S &= \int d\tau L_{particle}(\varphi(\xi(\tau))). \end{aligned} \tag{1}$$

Variation of this additional piece of action with respect to the field variable $\varphi(x)$ should yield the induced source term to the wave equation of the low frequency modes when averaged over the statistical distribution of the gas particles. The distribution can be derived from the solution of a Boltzmann equation describing the gas in the background field φ. Variation of (1) with respect to the particle trajectory provides the expression of the force exerted on the particle by the external field φ. This information is used in the collisionless Boltzmann-equation.

The effective particle Lagrangian for the one-component selfinteracting scalar fields described by the theory:

$$L[\varphi] = \frac{1}{2}(\partial_\mu\varphi)^2 - \frac{1}{2}m^2\varphi^2 - \frac{\lambda}{24}\varphi^4 \tag{2}$$

can be guessed intuitively by making use of the equation of motion for the real time Green function: $\Delta^>(x,y) = \langle\varphi(x)\varphi(y)\rangle$. Its Wigner-transform in the collisionless case fulfills [7]

$$(p_\mu\frac{\partial}{\partial X_\mu} + \frac{1}{2}\frac{\partial m^2(X)}{\partial X_\mu}\frac{\partial}{\partial p_\mu})\Delta^>(X,p) = 0, \tag{3}$$

where $m^2(X) = m^2 + (\lambda/2)\varphi^2(X)$, with φ representing the background at $X = (x+y)/2$. This equation suggests the relation

$$mF_\mu = \frac{1}{2}\partial_\mu m^2(X), \tag{4}$$

which is equivalent to

$$L_{particle} = -m[\varphi(x)]. \tag{5}$$

On the basis of this argument we propose for the classical effective action of an effective particle representation of the high-k modes of the one-component, self-interacting scalar field

$$\begin{aligned}\Delta S &= -\textstyle\int M_{loc}[\bar{\varphi}, \varphi(\xi(\tau))]d\tau,\\ M^2_{loc}[\bar{\varphi}, \varphi(x)] &= m^2 + \tfrac{\lambda}{2}(\bar{\varphi} + \varphi(x))^2,\end{aligned} \tag{6}$$

where m^2 is of negative sign, $\bar{\varphi}$ denotes the spontanously generated non-zero average background field, and $\varphi(x)$ is the amplitude of the low-k fluctuation field at x in the broken phase.

We propose to write the Boltzmann-equation for the effective particle with an x-independent mass m_{eff} (see below), which leads to a slight deviation from the kinetic equation derived by [7]. Its advantage is that the momentum variable of the one-particle distribution is defined through an x-independent relation. This definition gives slightly different $\mathcal{O}(\lambda^2)$ thermal mass to the low-frequency waves, but the damping coefficient remains unchanged. The necessary input into the derivation of the Boltzmann-equation is the equation for the kinetic momentum:

$$p_\mu = m_{eff}\frac{d\xi}{d\tau}, \quad m^2_{eff}(\bar{\varphi}) = m^2 + \frac{\lambda}{2}\bar{\varphi}^2. \tag{7}$$

The relevant equation can be found from the canonical (Euler-Lagrange) equation:

$$\begin{aligned}\tfrac{d}{d\tau}(M_{loc}[\bar{\varphi}, \varphi(x)]\dot{\xi}_\mu) &= \tfrac{dM_{loc}[\bar{\varphi},\varphi(x)]}{dV[\bar{\varphi},\varphi(x)]}\partial_\mu V[\bar{\varphi}, \varphi(x)],\\ V[\bar{\varphi}, \varphi(x)] &= \lambda(\bar{\varphi}\varphi + \tfrac{1}{2}\varphi^2), \quad x = \xi(\tau).\end{aligned} \tag{8}$$

From this equation, using explicitly the definition of M_{loc}, one can express the proper-time derivative of the kinetic momentum.

$$m_{eff}\frac{dp_\mu}{d\tau} = \frac{m^2_{eff}}{2M^2_{loc}}\Big[\partial_\mu V[\bar{\varphi}, \varphi(x)] - \frac{p_\mu}{m^2_{eff}}(p\cdot\partial)V[\bar{\varphi}, \varphi]\Big]. \tag{9}$$

The second term on the right hand side is missing from (4).

With help of (9) one writes the collisionless Boltzmann-equation for the gas of these particles:

$$(p\cdot\partial)f(x,p)+\frac{m_{eff}^2(\bar\varphi)}{2M_{loc}^2(\varphi,\bar\varphi)}\Big[\partial_\mu V[\bar\varphi,\varphi(x)]-\frac{p_\mu}{m_{eff}^2}(p\cdot\partial)V[\bar\varphi,\varphi]\Big]\frac{\partial f(x,p)}{\partial p_\mu}=0. \tag{10}$$

Its perturbative solution in the weak coupling limit $\lambda << 1$ is searched for in the form:

$$f(x,p)=f_0(p)+\lambda f_1(\varphi(x),p),\quad f_0(p)=\frac{1}{e^{\beta p_0}-1}. \tag{11}$$

(One may emphasize that the small parameter allowing iterative solution of Eq.(10) is hidden in V). The formal solution is easily found in full structural agreement with the result of [8]:

$$f_1(\varphi(x),p)=-\frac{1}{2}\Big[\frac{1}{(p\cdot\partial)}(\bar\varphi+\varphi(x))\partial_\mu\varphi(x)-\frac{p_\mu}{m_{eff}^2}(\bar\varphi\varphi(x)+\frac{1}{2}\varphi^2(x))\Big]\frac{df_0}{dp_\mu}. \tag{12}$$

3 Source density induced by field fluctuations

The variation of (6) with respect to φ provides the induced source term for the low-k fluctuations. For a number of particles moving on definite trajectories one has

$$j(x)=\frac{1}{2}\sum_i\int d\tau\Big(\frac{1}{M_{loc}[\bar\varphi,\varphi]}\frac{dV[\bar\varphi,\varphi(x)]}{d\varphi}\Big)\delta^{(4)}(x-\xi_i(\tau)). \tag{13}$$

Here the index i refers to the trajectory of the i-th particle. Statistical average over the full momentum space and in a small volume around the point x introduces the one-particle distribution

$$m_{eff}\int\frac{d^3p}{(2\pi)^3p_0}f(x,p)=\langle\int d\tau\sum_i\delta^{(4)}(\mathbf{x}-\xi_i(\tau))\rangle \tag{14}$$

and produces the following "macroscopic" source density:

$$j_{av}(x)=\frac{1}{2}\int\frac{d^3p}{(2\pi)^3p_0}\frac{m_{eff}(\bar\varphi)}{M_{loc}[\bar\varphi,\varphi]}\frac{dV[\bar\varphi,\varphi]}{d\varphi}f(x,p),\quad p_0^2=m_{eff}^2+\mathbf{p}^2. \tag{15}$$

The induced linear response to $\varphi(x)$ is found by retaining terms with linear functional dependence of j_{av} on φ. The contribution of the equilibrium distribution f_0 is easily found:

$$j_{av}^{(0)}(x) = \frac{\lambda}{2}\varphi(x)\int\frac{d^3p}{(2\pi)^3p_0}f_0(p_0)\left(1-\frac{\lambda\bar{\varphi}^2}{2m_{eff}^2}\right). \tag{16}$$

The $\mathcal{O}(\lambda^2)$ contribution is the result of the expansion of M_{loc} in the denominator of (15). This contribution to the thermal mass in the high-T limit gives the correct limiting value for the one-component scalar theory.

The terms induced by f_1 give rise to the following expression linear in φ:

$$\begin{aligned} j_{av}^{(1)}(x) &= \tfrac{\lambda^2\bar{\varphi}}{2}\int\tfrac{d^3p}{(2\pi)^3p_0}f_1(\varphi(x),p) \\ \sim -\tfrac{\lambda^2\bar{\varphi}^2}{4}\int\tfrac{d^3p}{(2\pi)^3p_0}\tfrac{1}{(p\cdot\partial)}\partial_0\varphi(x)\tfrac{df_0}{dp_0} &+ \tfrac{\lambda^2\bar{\varphi}^2\varphi(x)}{8\pi^2}\int_{m_{eff}}^{\infty}dp_0 f_0(p_0)\tfrac{1}{\sqrt{p_0^2-m_{eff}^2}}. \end{aligned} \tag{17}$$

The damping effect arises from the first term, while from the second another $\mathcal{O}(\lambda^2)$ contribution is obtained to the thermal mass.

The imaginary part of the linear response can be evaluated by the use of the principal value theorem, when the ϵ-prescription of Landau is applied to the $(p\cdot\partial)^{-1}$ operator in the first term on the right hand side of (17). For an explicit expression one may assume that φ represents an off-mass-shell fluctuation characterised by the 4-vector: $(\omega,\mathbf{k})$, that is $\varphi(x)=\varphi(k\cdot x)$. The evaluation of the integral is a not too difficult excercise. One is led to the following expression for the imaginary part of the linear response function:

$$\mathrm{Im}\Sigma = \frac{\lambda^2\bar{\varphi}^2}{16\pi}\frac{1}{e^{\beta m_eff/\sqrt{1-\omega^2/|\mathbf{k}|^2}}-1}\cdot\frac{\omega}{|\mathbf{k}|}\Theta(|\mathbf{k}|^2-\omega^2). \tag{18}$$

4 Discussion

In this note we have shown that in the broken phase of scalar theories Landau-type damping phenomenon occurs according to an effective classical kinetic theory. The clue to its existence is the presence of the $\sim\bar{\varphi}\varphi$ term in the fluctuating part of the local mass, which is responsible for the emergence of a linear source-amplitude relation with non-zero imaginary part. On the other hand the imaginary part of the self-energy has been computed earlier in the φ^4-theory using its real time quantum formulation [13], where "a term reminiscent of Landau damping" was found:

$$\begin{aligned} \mathrm{Im}\Sigma_{Landau} &= \frac{\lambda^2\bar{\varphi}^2 T}{16\pi|\mathbf{k}|}\ln\frac{1-e^{-\beta\omega_k^+}}{1-e^{-\beta\omega_k^-}}\Theta(|\mathbf{k}|^2-\omega^2), \\ \omega_{\mathbf{k}}^{\pm} &= \left|\frac{\omega}{2}\pm\frac{|\mathbf{k}|}{2}\sqrt{1-\frac{4M^2}{\omega^2-|\mathbf{k}|^2}}\right|. \end{aligned} \tag{19}$$

Here M is the temperature dependent mass parameter in the finite temperature quantum theory. With explicit calculation one may check that in the limit $\beta\omega, \beta|\mathbf{k}| << 1$ the expression of the damping rate calculated from (19) goes over into (18), under the assumption that m_{eff} of the classical theory is identified with M. In this way the classical theory provides a convincing argument clearly demonstrating that Landau damping is the correct interpretation of the result derived from quantum theory.

The extension of our discussion to the n-component scalar fields in the broken phase forces us to accept the choice (4), since for the Goldstone fields $m_{eff} = 0$. Then one can see by analysing the equations of motion of the Green-functions composed from the different field components, that with accuracy $\mathcal{O}(\lambda^2)$ one-particle distributions for each type of particles have their separate kinetic equation, decoupled from each other. Landau damping of the heavy ("Higgs") mode can be described in full analogy with the treatment of the present paper, writing for each particle a mechanical action of the form (6) with appropriate local mass expressions. Damping of the Goldstone modes is the result of the non-zero correlation between these and the "Higgs" modes. One can construct an additional non-local, velocity dependent mechanical action for the heavy particles describing this effect [17].

If one completes the kinetic theory of the pure non-Abelian gauge fields by appropriate scalar fields, an important generalisation of the present discussion can be made to the damping rates of gauge+Higgs systems, where also the interpretation of lattice simulations [14] represents a non-trivial challenge [15]. Some results concerning the hard thermal loop effects on thermal masses of the gauge bosons have been obtained recently in [16]. An approach based on the coupled kinetic system is presently under study [17].

The authors acknowledge informative discussions on the subject of the paper with A. Jakovác and P. Petreczky. This work has been supported by the research contract OTKA-T22929.

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